The Mathematics Department
Presents
The Problem of the Month
March 2004
Heads Will Roll

The Problem:
Imagine that you were to roll a quarter around the inside of a circle whose
diameter was precisely twice that of a quarter’s. How many complete revo-
lutions would George Washington’s head make when the quarter returned to
its initial position? What if you were to roll a quarter around the outside of
the same circle? How many complete revolutions would George’s head make this time?

A Solution:
As the diameter of the circle was exactly twice that of a quarter, the circum-
ference of the circle would be exactly twice that of the quarter’s, as well.
This is because the formula for the circumference of a circle is \( \pi D \), where \( D \) is the diameter. If the diameter is doubled, then the circumference is dou-
bled. Now imagine the circle to be cut and laid out straight. The length of
this line segment would be precisely the circumference of the circle.
Rolling the quarter along the line from beginning to end would result in the quarter making exactly two complete revolutions. Thus, if the circle were a line, George’s head would make exactly two revolutions. Now, rolling the line back up into a circle subtracts one of the two revolutions for the quarter inside circle and adds one revolution to the quarter outside the circle. Thus the inside quarter makes exactly one revolution and the outside quarter makes exactly three revolutions.