The Mathematics Department
Presents
The Problem of the Month
October 2004

A Boy or a Girl?

The Problem: Mrs. Smith and Mrs. Jones both have two children. None of the children are twins. Mrs. Smith tells you that her older child is a boy. Mrs. Jones tells you that (at least) one of her two children is a boy. In each case, what is the likelihood that the other child is a girl? Are your answers the same in both cases? Different? Why or why not?

A Solution: Without any additional constraints, one can consider four possible ways for two children to be situated within a family. They are:

1) Boy / boy       2)  Boy / girl       3) Girl / boy       4) Girl / girl

Here, we have listed the four possible arrangements with the upper case gender denoting the older child. These four arrangements are all equally likely.

Now Mrs. Smith tells us that her older child is a boy. She is telling us that her family is arranged according to configuration 1 or 2. Both of these are equally likely. In one of the two, the other child is a girl (#2) and so the probability that Mrs. Smith’s other child is a girl is $\frac{1}{2}$.

Mrs. Jones, on the other hand, tells us that at least one of her two children is a boy. In essence, she is informing us that her family is arranged according to configuration 1, 2 or 3. Configuration 4 is ruled out. As all three of the configurations in contention are equally likely and two of the three (#2 and #3) have a girl as the other child, the probability that Mrs. Jones’ other child is a girl is $\frac{2}{3}$. 