The Mathematics Department Presents
The Problem of the Month
April 2003
Plenty of Nothin’

For a positive integer, $N$, we define $N!$ (read N factorial) to be the product of all the integers from 1 to $N$. Thus:

$$N! = 1 \times 2 \times 3 \times \ldots \times N$$

It is clear that $N!$ will end in a zero if $N$ is larger or equal to 5. For big values of $N$, $N!$ will end in many, many zeros. How many zeros will $75!$ end in? Justify your answer.

A Solution: The number of zeros a number ends in is precisely the highest power of 10 that divides it. A number ending in 2 zeros, for example, is divisible by 100 but not 1,000. Thus, we need to find the highest power of 10 that divides $75!$. Clearly, every time we multiply by a factor of 2 and a factor of 5, we get a factor of 10. Every other factor in $75!$ is even, so we have many, many factors of 2. Every $5^{th}$ factor of $75!$ will be divisible by 5. There are 15 such factors: 5, 10, 15, 20, …., 65, 70, 75. Note, however, that three of these factors (25, 50 and 75) contribute two factors of 5, each. Consequently, there are $15 + 3 = 18$ factors of 5 in $75!$. As there are far more than 18 factors of 2 in $75!$, we may conclude that there are 18 zeros at the end of $75!$

Congratulations to Andrew Scotti and Pat McDermott, both of Christ the King Regional High School! They were the only correct solvers.