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Gauss Goes to the Pizza Parlor: True Facts in a **False Story**

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True Facts in a False Story

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David Seppala-Holtzman



ine-year-old Carl Friedrich Gauss came running home from school, all excited. "Father! Father! I had the most wonderful day. My teacher said I was a genius!

"He gave us the task of summing the numbers from 1 to 100 as a punishment for our misbehavior, and I figured out a simple way to get the answer. I wrote out the numbers in a row from left to right and then again directly below that row from right to left," he said, as he wrote the digits for his father. (See figure 1.)

"Adding the two rows, column-wise, we get 100 copies of 101, yielding a sum of 10,100. As each

Figure 1. Gauss's calculation of the sum of the first 100 numbers.

1	H	F (2	+	•	3	+	•••	+		99	+	1	00
100	+	9	9	+	9	98	+	•••		+	2	+	F	1
101	+	10	1	+	10	21	+		+	1	01	+	1	01

number appears twice, we simply divide by 2 to get the correct answer of 5,050."

"My son, you *are* such a clever lad. We should celebrate. Let us go out for pizza."

"Oh yes, father, let's!"

At the Pizza Parlor

After the Gausses got their slices, they searched for cutlery in vain. The proprietor apologized and told them that he had no clean knives or forks, and they should use their hands.

Carl picked up his slice and, to his dismay, the tip drooped down. He was afraid that in his attempt to get the slice into his mouth, he would end up with toppings all down his shirt. After a moment's thought, he curled the crust end into a "U" and, lo and behold, the tip stood straight out, offering up a cheesy reward.

"Ah! You are so clever! I wonder why that works," his father said offhandedly, not expecting an answer. But the young Gauss knew why, and, with his mouth full of pizza, began explaining.

"Let's start with a curve in the plane and a point, *p*, on that curve." Carl drew an arbitrary curve on his napkin, as shown in figure 2.

"There are infinitely many circles whose tangent line at *p* agrees with the tangent line to the curve at that point. One of these circles best approximates the curve. (I shall leave the definition of *best* for another day.) It is called the *osculating circle*, which comes from the Latin for 'kissing," he said, blushing.

"The reciprocal of the radius of the osculating circle is called the *curvature* at *p*. If the osculating circle has infinite radius—for example, if the curve is a straight line—then the curvature is zero.

"Now, suppose we go up a dimension, so we have a surface in space and a point, *p*, on the surface." Carl drew the ellipsoid and the saddle-shaped surface on another napkin (see figure 3).

Figure 2. A curve and the osculating circle at *p*.

Figure 3. The principal curvatures for an ellipsoid have the same sign and they have opposite signs for a saddle-shaped surface.



"There are infinitely many planes containing the normal vector to the surface at p, and each cuts the surface in a curve. Each of these curves has a curvature at p. We take the curvature to be positive if the circle is on the same side as the normal vector and negative otherwise. The maximal and minimal curvatures are called the *principal curvatures*. If these values are not the same, as they would be on a sphere, then the planes that gave rise to these values are orthogonal—a wonderful result that the great Leonhard Euler proved not long ago.

"Let's multiply the principal curvatures at *p* to get a single curvature value. Notice that this value is positive if the osculating circles for the principal curves lie on the same side of the surface and is negative if they lie on opposite sides." Little did either of them know, this value would later be known as the *Gaussian curvature* at *p*.

"For example, the curvature value for a sphere of radius r is $1/r^2$ at every point, and for a general ellipsoid it is positive everywhere. A flat plane has curvature zero, as does a cylinder. A saddle has negative curvature.

"One remarkable thing I've noticed is that this curvature value does not change if one bends the surface without stretching it!" This amazing result of Gauss's would later be known as the *Theorem Egregium*, which is Latin for the "Remarkable Theorem."

"Now," Gauss said, picking up another slice of the pizza, "because cooked dough is not stretchy, a pizza slice has the same curvature Figure 4. The curvature of a pizza slice is zero.



whether the crust is curled into a "U" or not. And because a pizza slice can sit flat on a plate, it must have curvature zero. Thus, a curled slice must also have curvature zero. This means that the principal curve orthogonal to the curled crust, which is just the radial line from crust to tip, must have principal curvature zero. In other words, the tip of the slice sticks straight out!"

And with that, Carl bent the crust into a "U" and took a bite.

His father sat in silent amazement as his son explained all of this. "Your teacher was right. You are a genius. You deserve, not only another slice of pizza but, perhaps an entire pie!" Then he added with an arched eyebrow, "Or should I say P-I, π ?" Carl rolled his eyes and groaned at the dad joke. "Speaking of π ," the young boy said, "I've been thinking about probability, and π makes a surprising appearance in this certain bell-shaped distribution. . ."

David Seppala-Holtzman received his doctorate from the University of Oxford in 1976. After five years in the Netherlands, he joined the faculty of St. Joseph's College in Brooklyn and has been there ever since. His European and New York sensibilities come into conflict whenever he eats pizza, as he prefers using a knife and fork, generating scorn and laughter from the locals.

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Which is the Odd One Out?

JEREMIAH FARRELL

π , PI, PIE, TURNIP

The solution to this puzzle is at the bottom of the page.

Jeremiah Farrell is an emeritus professor of mathematics at Butler University. He is known for designing Will Shortz's favorite New York Times crossword puzzle, the 1996 "Election Day."

Because π can be removed as the only non-word, and TURNIP can be removed as the one with a different pronunciation, and PIE can be removed as the only word not representing the 16th letter of the Greek alphabet (TURNIP = TURN "IP" = PI), the odd one out is PI because it cannot be removed.

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