Principles of Corporate Finance

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CHAPTER 5

Time Value of Money

"Time is money… really "

McGraw-Hill/Irwin

Time Value of Money

Money has a time value. It can be expressed in multiple ways:

A dollar today held in savings will grow.

A dollar received in a year is not worth as much as a dollar received today.

To make meaningful comparisons we must adjust for time.

Future Values

Future Value - Amount to which an investment will grow after earning interest.

Compound Interest - Interest earned on interest.

Simple Interest - Interest earned only on the original investment.

Future Values

Compound Interest

Interest earned at a rate of 6% for five years on the previous year's balance.

	Today	Future Years				.
		1	*2*	*3*	*4*	*5*
Interest Earned		6.00	6.36	6.74	7.15	**7.57**
Value	100	106.00	112.36	119.10	126.25	**133.82**

Value at the end of Year 5 = $133.82

Future Values

Future Value of $100 = FV

$$FV = \$100 \times (1+r)^t$$

$$FV = PV \times (1+r)^t$$

Future Values

$$FV = PV \times (1+r)^t$$

<u>*Example - FV*</u>

What is the future value of $100 if interest is compounded annually at a rate of 6% for five years?

$$FV = \$100 \times (1+.06)^5 = \$133.82$$

Simple Interest

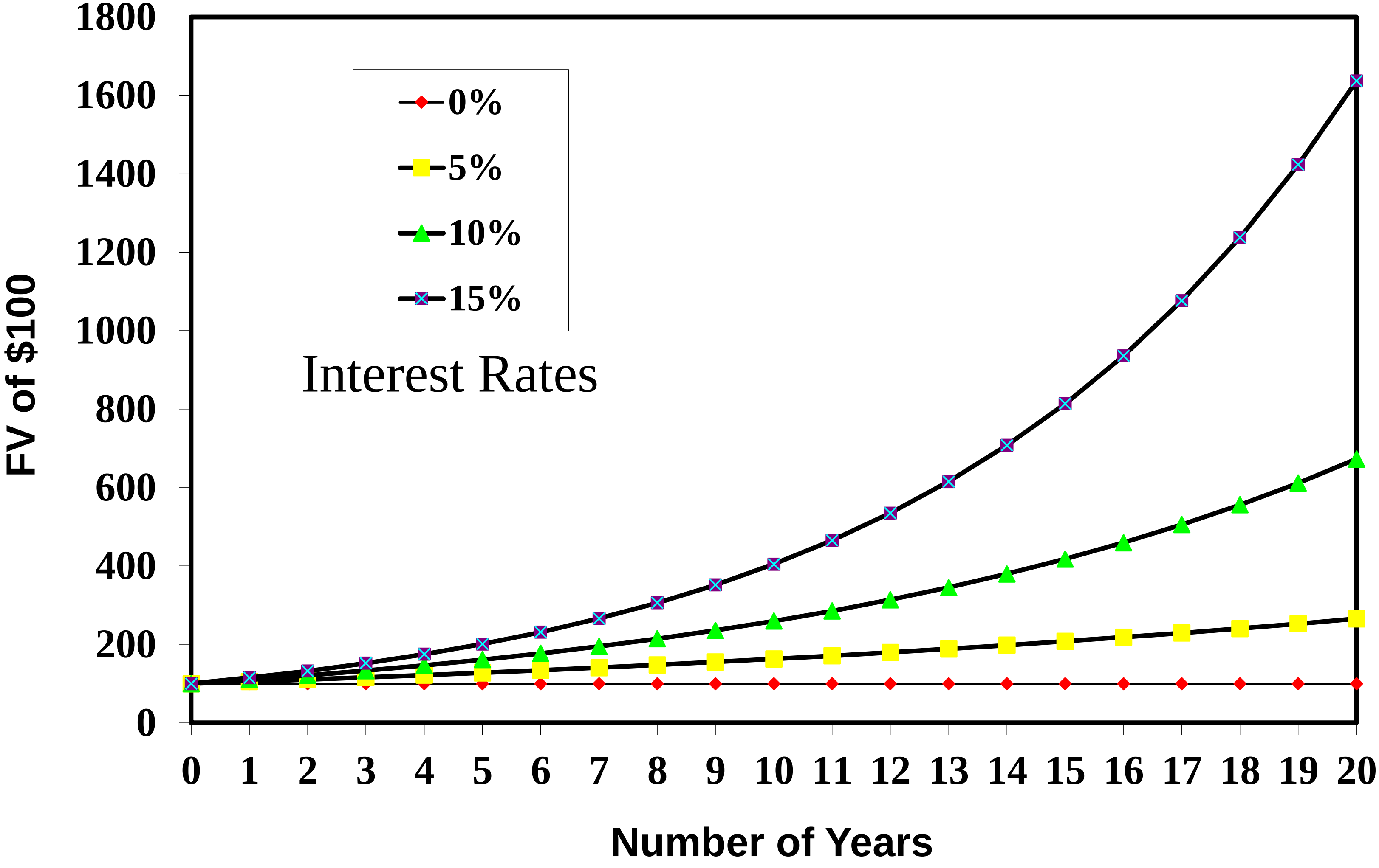
Simple Interest

Interest earned at a rate of 6% for five years on a principal balance of $100.

	Today	Future Years				
		1	2	3	4	5
Interest Earned		6	6	6	6	6
Value	100	106	112	118	124	130

Value at the end of Year 5 = $130

Future Values with Compounding



Manhattan Island Sale

Peter Minuit bought Manhattan Island for $24 in 1626.
Was this a good deal?

To answer, determine $24 is worth in the year 2020, compounded at 8%.

N=394
I=8
PV=24
PMT=0

$$FV = \$24 \text{ x } (1+.08)^{394}$$
$$= \$354.1 \text{ trillion}$$

FYI - The value of Manhattan Island land is well below this figure.

Present Value

$$FV = PV \text{ x } (1+r)^t$$

$$PV = \frac{\text{Future Value after t periods}}{(1+r)^t}$$

Present Value

Example

If you can earn 8% on your money, how much money should you set aside today in order to buy a computer that will cost $3000 in two years?

$$PV = \frac{3000}{(1.08)^2} = \$2,572$$

FV=3000
N=2
I=8
PMT=0

Present Value

Present Value

Value today of a future cash flow.

Discount Rate

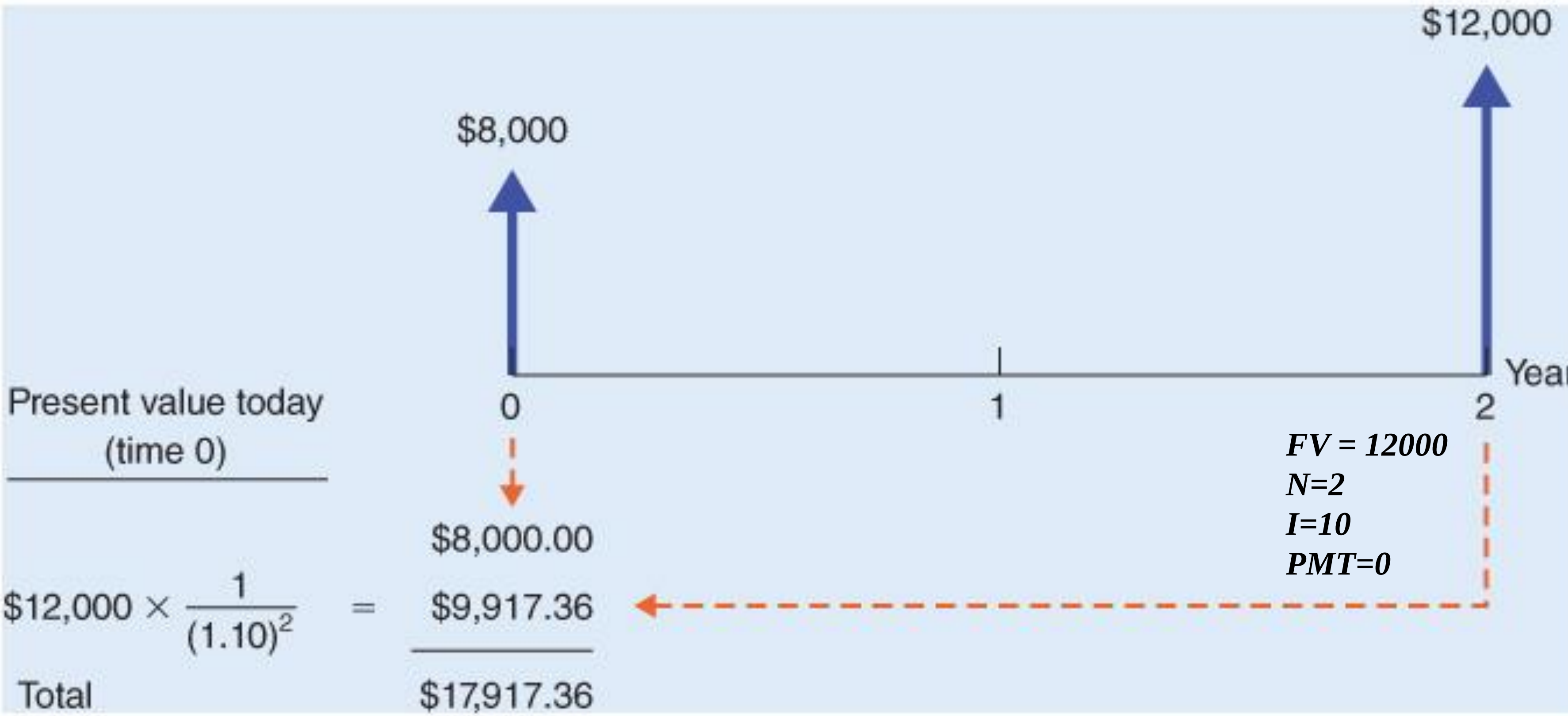
Interest rate used to compute present values of future cash flows.

Discount Factor

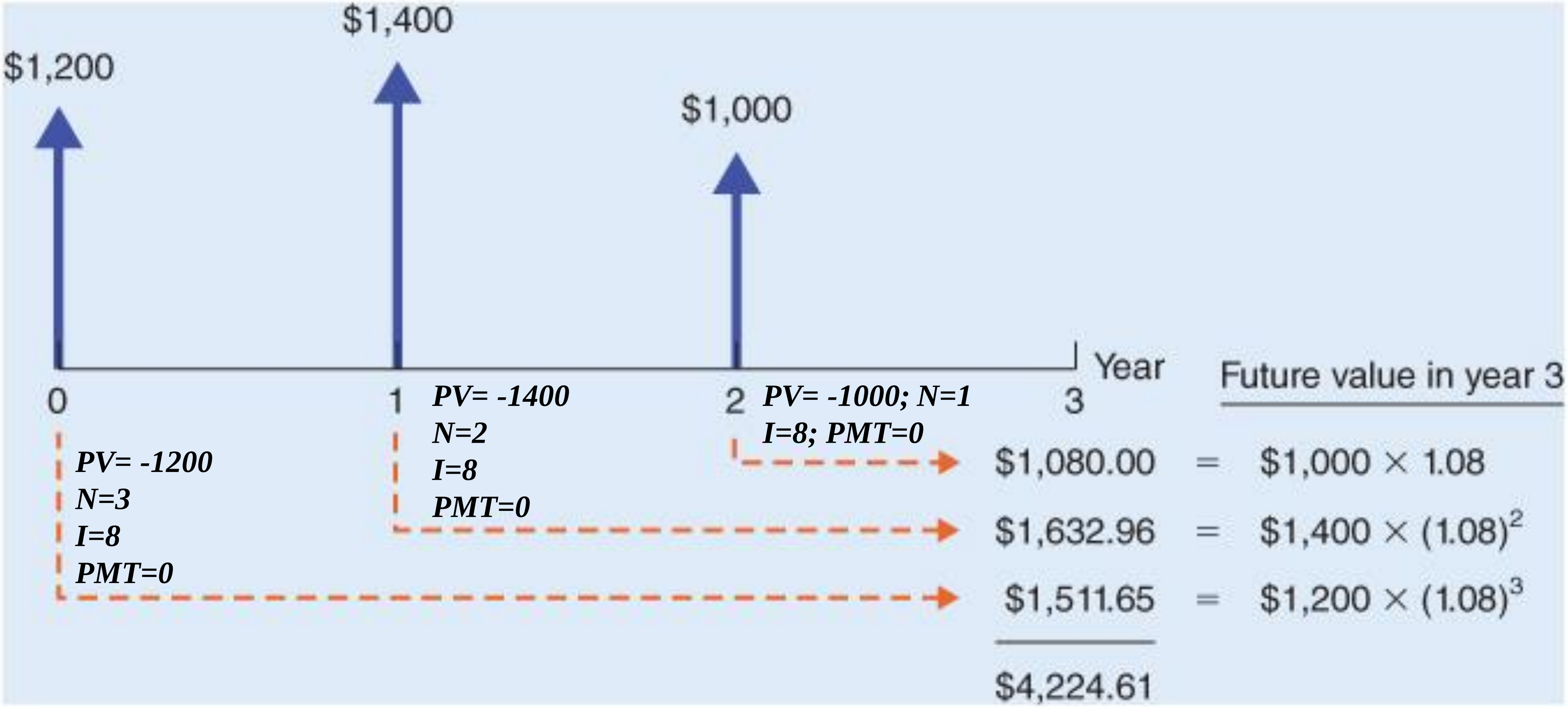
Present value of a $1 future payment.

$$DF = \frac{1}{(1+r)^t}$$

Present Value



FV of Multiple Cash Flows

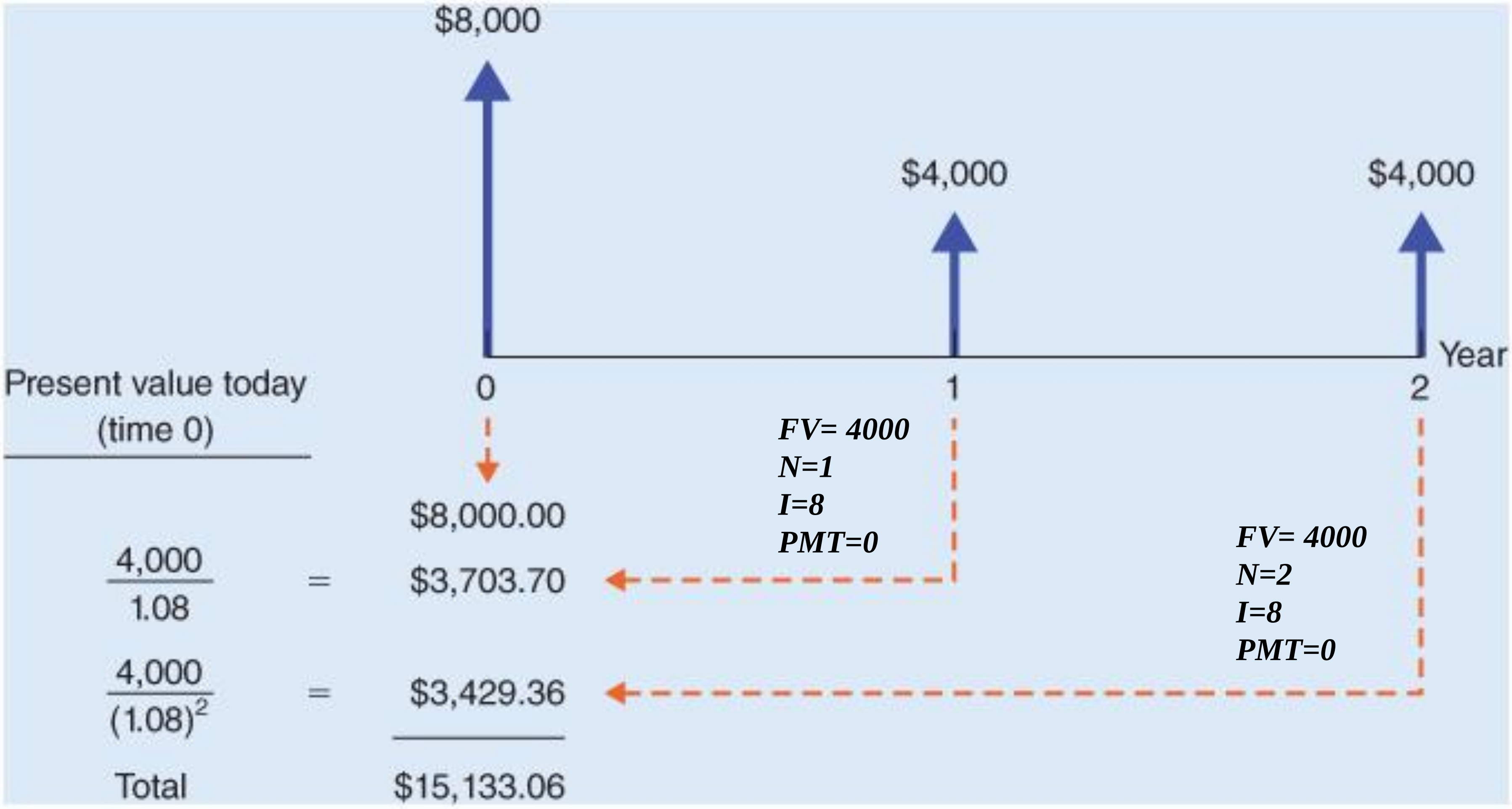


PV of Multiple Cash Flows

- PVs can be added together to evaluate multiple cash flows.

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots.$$

PV of Multiple Cash Flows



Simplifications

Perpetuity

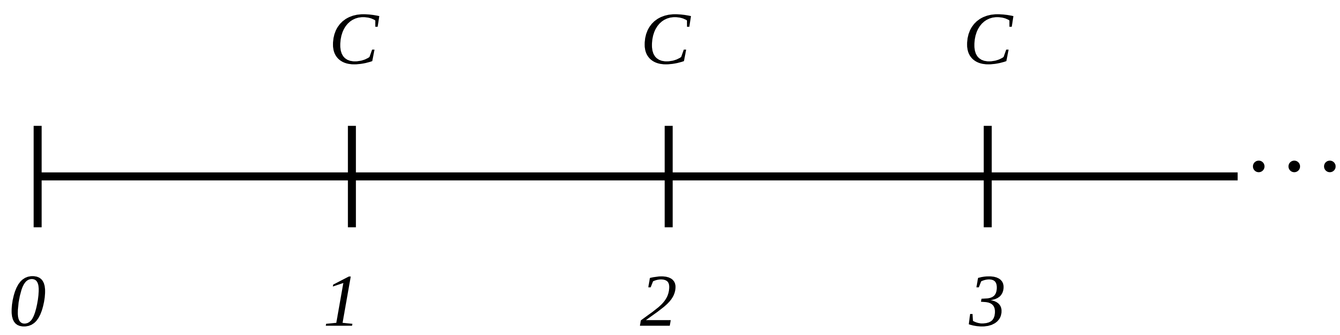
- A constant stream of cash flows that lasts forever.

Annuity

- A stream of constant cash flows that lasts for a fixed number of periods.

Perpetuity

A constant stream of cash flows that lasts forever.



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

The formula for the present value of a perpetuity is:

$$PV = \frac{C}{r}$$

C = cash payment
r = interest rate

Perpetuities & Annuities

Perpetuity

In order to create an endowment, which pays $100,000 per year, forever, how much money must be set aside today if the rate of interest is 10%?

$$PV = \frac{100{,}000}{.10} = \$1{,}000{,}000$$

Perpetuities & Annuities

Example - continued

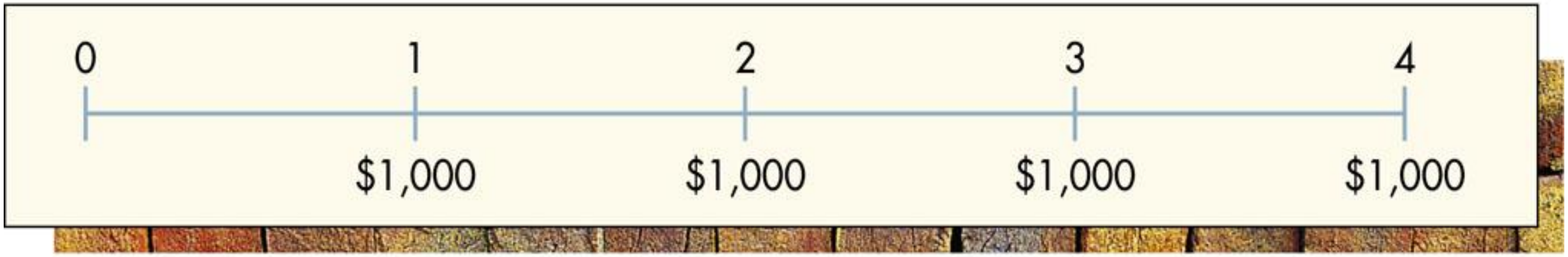
If the first perpetuity payment will not be received until three years from today, how much money needs to be set aside today?

$$PV = \frac{100{,}000}{.10} \text{ x } \frac{1}{(1+.10)^3}$$

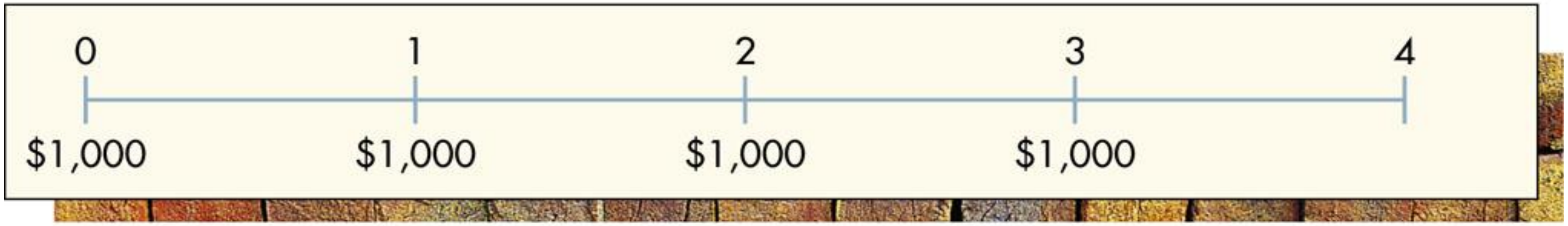
$$= \$751{,}315$$

Annuities

Ordinary Annuity



Annuity Due



Valuing an Annuity

			Cash Flow				
Year	1	2	3	4	5	6…	Present Value
Perpetuity A	$1	$1	$1	$1	$1	$1…	$\frac{1}{r}$
Perpetuity B				$1	$1	$1…	$\frac{1}{r(1+r)^3}$
Three Year annuity	$1	$1	$1				$\frac{1}{r}-\frac{1}{r(1+r)^3}$

Perpetuities & Annuities

PV of Annuity Formula

$$PV = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

C = cash payment

r = interest rate

t = Number of years cash payment is received

Perpetuities & Annuities

PV Annuity Factor (PVAF) - The present value of $1 a year for each of t years.

$$PVAF = \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

Perpetuities & Annuities

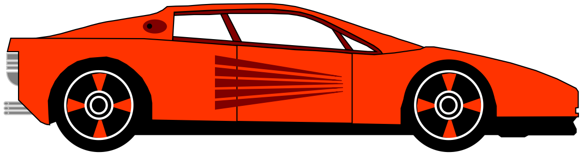
FV= 0
N=3
I=10
PMT=8000

Annuity

You are purchasing a car. You are scheduled to make 3 annual installments of $8,000 per year. Given a rate of interest of 10%, what is the price you are paying for the car (i.e. what is the PV)?

$$PV = 8{,}000\left[\frac{1}{.10} - \frac{1}{.10(1+.10)^3}\right]$$

$$PV = \$19{,}894.82$$



Mortgage Amortization Table

Month	Outstanding Balance	Payment	Interest Paid	Principal Paid
1	$200,000.00	$1609.25	$1500.00	$109.25
2	199,890.75	1609.25	1499.18	110.07
3	199,780.68	1609.25	1498.36	110.89
4	199,669.79	1609.25	1497.52	111.73
Etc.				

Effective Interest Rates

Annual Percentage Rate - Interest rate that is annualized using simple interest.

Effective Annual Interest Rate - Interest rate that is annualized using compound interest.

Effective Interest Rates

Example:

Given a monthly rate of 1%, what is the Effective Annual Rate(EAR)? What is the Annual Percent-age Rate (APR)?

$$EAR = (1+.01)^{12} - 1 = r$$

$$EAR = (1+.01)^{12} - 1 = .1268 \text{ or } 12.68\%$$

$$APR = .01 \times 12 = .12 \text{ or } 12.00\%$$

Compounding Frequency

Compounding	Periods		Per Period		Effective
Period	Per Year	APR	Interest Rate	Growth Factor	Annual Rate
1 year	1	6%	6%	1.06	6.0000%
Semiannually	2	6%	3	$1.03^2 = 1.0609$	6.0900
Quarterly	4	6%	1.5	$1.015^4 = 1.061364$	6.1364
Monthly	12	6%	0.5	$1.005^{12} = 1.061678$	6.1678
Weekly	52	6%	0.11538	$1.0011538^{52} = 1.061800$	6.1800
Daily	365	6%	0.01644	$1.0001644^{365} = 1.061831$	6.1831
Continuously	---	6%	e^{APR}	$2.718^{.06} = 1.061837$	6.1837

- FYI: The general formula for the future value of an investment compounded continuously over many periods can be written as:

$$FV = PV \times e^{rt}$$

Inflation

Inflation - Rate at which prices as a whole are increasing.

Nominal Interest Rate - Rate at which money invested grows.

Real Interest Rate - Rate at which the purchasing power of an investment increases.